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ON EQUATIONS OF A ONE-VELOCITY HETEROGENEOUS MEDIUM

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An analysis is made of equations of the one-velocity model of a heterogeneous mixture; in them, the internal interfractional-interaction forces and heat- and mass-exchange processes are allowed for. The characteristic equations and relations along the characteristic directions are found. It is shown that equations of the medium's model, in which the interfractional-interaction forces are allowed for, belong to a hyperbolic type. A number of finite-difference and finite-volume schemes meant for integration of the model's equations are considered.

Keywords: *one-velocity multicomponent medium, hyperbolic systems of the nondivergent form, numerical modeling.*

Introduction. The one-velocity model of a heterogeneous medium with one pressure is widely used in modeling various shock-wave problems in foamy and bubble liquids and water-saturated grounds and in other heterogeneous systems where the one-velocity approximation is allowed. Gasdynamic equations considered in combination with the barotropic equation of state which adequately describes the behavior of heterogeneous mixtures were used in the early models. We note the works in which it has been proposed that the complex properties of heterogeneous systems described by the gasdynamic equations, too, be approximated by assigning a variable adiabatic exponent of the mixture (see [1]).

In addition to the gasdynamic equations (for the entire mixture), today's models of a one-velocity medium contain the equations for individual components of the mixture. The corresponding equations can be obtained from the multivelocity-continuum model (see [2]) in which it is necessary to set the component velocities equal to each other and to allow for pressure equilibrium. In this case the densities of interfractional-interaction forces $\mathbf{f}_{ij}$ must formally be set equal to zero, since they are in proportion to the difference in the velocities of fractions marked by i and j. The model of a one-velocity medium, obtained using this approach, has been investigated in [3]. It has turned out that for mixtures containing two or more compressible fractions, the Cauchy problem is correct only in a limited velocity range.

In the present work, we use the one-velocity model of a medium, constructed on a different principle (see [4]). The coincidence of the fraction velocities is initially built into the model. Nonzero internal forces of interfractional interaction are introduced to ensure equal accelerations for each different-density fraction. With such an approach the Cauchy problem turns out to be correct for any number of fractions in the mixture and an arbitrary velocity of motion. Furthermore, for mixtures in which the above forces are allowed for, the corresponding systems of equations for the adiabatic case, unlike the equations of the model from [3], are hyperbolic for any values of the parameters. Also, we note that in the model proposed, the properties of the mixture are determined only by the constants of individual constituent fractions of the heterogeneous medium.

Differential Equations of the Model. Let us consider a one-velocity n-component mixture with the first m compressible fractions. We write equations describing flow of a heterogeneous medium, in which the interfractional-interaction forces are allowed for (see [4]):

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{u}) = 0 , \quad \rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\, \mathbf{u} \right] + \operatorname{grad} p = \mathbf{F} ,$$

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$$\frac{\partial}{\partial t}\left[\rho\left(\varepsilon+\frac{1}{2}\,|\mathbf{u}|^2\right)\right]+\operatorname{div}\left[\rho\left(\varepsilon+\frac{1}{2}\,|\mathbf{u}|^2+\frac{p}{\rho}\right)\mathbf{u}+\sum_{i=1}^{n}\alpha_i\mathbf{W}_i\right]=\mathbf{F}\cdot\mathbf{u}\,;$$

$$\frac{\partial\alpha_i\rho_i^0}{\partial t}+\operatorname{div}\left(\alpha_i\rho_i^0\mathbf{u}\right)=\sum_{k=1}^{n(k\neq i)}J_{ik}\,,$$

$$\rho_i\left(\frac{\partial\varepsilon_i}{\partial t}+(\mathbf{u}\cdot\nabla)\,\varepsilon_i\right)+\frac{\alpha_i p}{\rho_i}\left[\sum_{k=1}^{n(k\neq i)}J_{ik}-\left(\frac{\partial\rho_i}{\partial t}+(\mathbf{u}\cdot\nabla)\,\rho_i\right)\right]+\operatorname{div}\,(\alpha_i\mathbf{W}_i)$$

$$=\sum_{k=1}^{n(k\neq i)}(R_{ik}+Q_{ik})-\left(\varepsilon_i-\frac{1}{2}\,|\mathbf{u}|^2\right)\sum_{k=1}^{n(k\neq i)}J_{ik}\,,\quad i=1,\,...,\,m-1\,;$$

$$\frac{\partial\alpha_j}{\partial t}+\operatorname{div}\,(\alpha_j\mathbf{u})=\frac{1}{\rho_j^0}\sum_{k=1}^{n(k\neq j)}J_{jk}\,,\quad j=m+1,\,...,\,n\,,\tag{1}$$

where

$$\varepsilon=\frac{1}{\rho}\sum_{i=1}^{n}\rho_i\varepsilon_i\,.\tag{2}$$

The behavior of the compressible fractions is described by the calorific equations of state

$$\varepsilon_i=\varepsilon_i\left(p,\,\rho_i^0\right),\tag{3}$$

therefore, expression (2) takes the form

$$\varepsilon=\frac{1}{\rho}\left(\sum_{i=1}^{m}\alpha_i\rho_i^0\varepsilon_i+\sum_{j=m+1}^{n}\alpha_j\rho_j^0\varepsilon_j\right).\tag{4}$$

Allowing for the relations $\sum_{i=1}^{n}\alpha_i = 1$ and $\rho = \sum_{i=1}^{n}\alpha_i\rho_i^0$ and eliminating α_m and ρ_m^0 from (4), we have

$$\varepsilon=\varepsilon\left(\rho,\,p,\,\alpha_1,\,\rho_1^0,\,...,\,\alpha_{m-1},\,\rho_{m-1}^0,\,\alpha_{m+1},\,...,\,\alpha_n\right).\tag{5}$$

The temperature of the ith fraction can be found from the thermal equation of state of the component

$$T_i=T_i\left(p,\,\rho_i^0\right).\tag{6}$$

If we use, in describing the behavior of the compressible components of the mixture, the equations of state

$$\varepsilon_i=\frac{p-c_{*i}^2\left(\rho_i^0-\rho_{*i}\right)}{\rho_i^0\,(\gamma_i-1)}\,,\tag{7}$$

expression (5) for the n-component mixture with the first m compressible fractions takes the form

$$\varepsilon = \frac{1}{\rho}\left[\sum_{i=1}^{m-1}\alpha_i\left(pB_{im} - d_{im}\rho_i^0 + b_{im}\right) + pB_m + b_m + \sum_{j=m+1}^{n}\alpha_j\rho_j^0\varepsilon_j\right] - d_m \,. \tag{8}$$

Here $B_i = 1/(\gamma_i - 1)$, $B_{im} = B_i - B_m$, $d_i = c_{*i}^2 B_i$, $d_{im} = d_i - d_m$, $b_i = d_i\rho_{*i}$, and $b_{im} = b_i - b_m$. The equation of state (7) for the ith fraction can be rewritten as follows:

$$\varepsilon_i = \frac{pB_i + b_i}{\rho_i^0} - d_i \,. \tag{9}$$

Let us consider the adiabatic version of the model in whose equations we also drop mass forces. If we eliminate the variables ε and ε_i from system (1), using relations (3) and (5) for this purpose, we obtain the system

$$\frac{\partial\rho}{\partial t} + \operatorname{div}(\rho\mathbf{u}) = 0\,, \qquad \frac{\partial\mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} + \frac{1}{\rho}\operatorname{grad} p = 0\,,$$

$$A_1\left(\frac{\partial p}{\partial t} + (\mathbf{u}\cdot\nabla)\,p\right) + \left(A_2 - \frac{p}{\rho^2}\right)\left(\frac{\partial\rho}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho\right)$$

$$+\sum_{i=1}^{m-1}\left[A_{i+m+1}\left(\frac{\partial\alpha_i}{\partial t} + (\mathbf{u}\cdot\nabla)\,\alpha_i\right) + A_{i+2}\left(\frac{\partial\rho_i^0}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho_i^0\right)\right] + \sum_{j=m+1}^{n} A_{j+m}\left(\frac{\partial\alpha_j}{\partial t} + (\mathbf{u}\cdot\nabla)\,\alpha_j\right) = 0\,;$$

$$\frac{1}{\rho_i^0}\left(\frac{\partial\rho_i^0}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho_i^0\right) + \frac{1}{\alpha_i}\left(\frac{\partial\alpha_i}{\partial t} + (\mathbf{u}\cdot\nabla)\,\alpha_i\right) - \frac{1}{\rho}\left(\frac{\partial\rho}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho\right) = 0\,, \tag{10}$$

$$K_i\left(\frac{\partial p}{\partial t} + (\mathbf{u}\cdot\nabla)\,p\right) + L_i\left(\frac{\partial\rho_i^0}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho_i^0\right) - \frac{1}{\alpha_i}\left(\frac{\partial\alpha_i}{\partial t} + (\mathbf{u}\cdot\nabla)\,\alpha_i\right) = 0\,, \quad i = 1, ..., m-1\,;$$

$$\frac{1}{\alpha_j}\left(\frac{\partial\alpha_j}{\partial t} + (\mathbf{u}\cdot\nabla)\,\alpha_j\right) - \frac{1}{\rho}\left(\frac{\partial\rho}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho\right) = 0\,, \quad j = m+1, ..., n\,.$$

Here we have introduced the following notation: $A_1 = \dfrac{\partial\varepsilon}{\partial p}$, $A_2 = \dfrac{\partial\varepsilon}{\partial\rho}$, $A_{i+2} = \dfrac{\partial\varepsilon}{\partial\rho_i^0}$, ..., $A_{i+m+1} = \dfrac{\partial\varepsilon}{\partial\alpha_i}$, $K_i = \dfrac{\rho_i^0}{p}\dfrac{\partial\varepsilon}{\partial p}$, $L_i = \dfrac{\rho_i^0}{p}\dfrac{\partial\varepsilon_i}{\partial\rho_i^0} - \dfrac{1}{\rho_i^0}$, $i = 1, ..., m-1$, and $A_{j+m} = \dfrac{\partial\varepsilon}{\partial\alpha_j}$, $j = m+1, ..., n$. Allowing for the relations for individual components, we rewrite the energy equation for the entire mixture from system (10) as follows:

$$\left(\frac{\partial p}{\partial t} + (\mathbf{u}\cdot\nabla)\,p\right) - c^2\left(\frac{\partial\rho}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho\right) = 0\,,$$

where

$$c = \left\{\frac{\dfrac{p}{\rho} - \rho\dfrac{\partial\varepsilon}{\partial\rho} - \displaystyle\sum_{i=1}^{m-1}\left[\frac{p}{\rho_i^0}\frac{\partial\varepsilon}{\partial\rho_i^0}\left(\frac{\partial\varepsilon_i}{\partial\rho_i^0}\right)^{-1} + \alpha_i\frac{\partial\varepsilon}{\partial\alpha_i}\left(1 - \frac{p}{(\rho_i^0)^2}\left(\frac{\partial\varepsilon_i}{\partial\rho_i^0}\right)^{-1}\right)\right] - \sum_{j=m+1}^{n}\alpha_j\frac{\partial\varepsilon}{\partial\alpha_j}}{\rho\left[\dfrac{\partial\varepsilon}{\partial p} + \displaystyle\sum_{i=1}^{m-1}\frac{\partial\varepsilon_i}{\partial p}\left(\frac{\partial\varepsilon_i}{\partial\rho_i^0}\right)^{-1}\left(\frac{\alpha_i}{\rho_i^0}\frac{\partial\varepsilon}{\partial\alpha_i} - \frac{\partial\varepsilon}{\partial\rho_i^0}\right)\right]}\right\}^{1/2}. \tag{11}$$

In the particular case of mixtures the individual properties of whose components are described with the equations of state of the form (9), expression (11) will take the form

$$c = \left\{ \frac{b_m + p \left[1 + B_m - \sum_{i=1}^{m-1} \frac{\alpha_i \, (b_{im} + pB_{im})}{b_i + pB_i} \right]}{\rho \left[B_m + \sum_{i=1}^{m-1} \frac{\alpha_i \, (b_m B_i - b_i B_m)}{b_i + pB_i} \right]} \right\}^{1/2} . \qquad (12)$$

Formula (12) yields that the velocity of sound in the mixture is independent of the properties of incompressible fractions.

Thus, we have the following system of quasilinear equations:

$$\frac{\partial \rho}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho + \rho \operatorname{div} \mathbf{u} = 0 \,, \quad \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} + \frac{1}{\rho} \operatorname{grad} p = 0 \,, \quad \frac{\partial p}{\partial t} + (\mathbf{u}\cdot\nabla)\, p + \rho c^2 \operatorname{div} \mathbf{u} = 0 \,;$$

$$\frac{\partial \rho_i^0}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho_i^0 + \rho_i^0 G_i \operatorname{div} \mathbf{u} = 0 \,, \quad \frac{\partial \alpha_i}{\partial t} + (\mathbf{u}\cdot\nabla)\,\alpha_i + \alpha_i\,(1 - G_i) \operatorname{div} \mathbf{u} = 0 \,, \quad i = 1, ..., m-1 \,;$$

$$\frac{\partial \alpha_j}{\partial t} + (\mathbf{u}\cdot\nabla)\,\alpha_j + \alpha_j \operatorname{div} \mathbf{u} = 0 \,, \quad j = m+1, ..., n \,, \qquad (13)$$

where

$$G_i = \frac{1 - \rho c^2 K_i}{1 + \rho_i^0 L_i} = \frac{1}{\rho_i^0} \left(\frac{\partial \varepsilon_i}{\partial \rho_i^0} \right)^{-1} \left(\frac{p}{\rho_i^0} - \rho c^2 \frac{\partial \varepsilon_i}{\partial p} \right).$$

For one-dimensional flows, we rewrite system (13) in vector-matrix form

$$\frac{\partial \mathbf{U}}{\partial t} + \mathrm{A} \frac{\partial \mathbf{U}}{\partial r} + \mathbf{B} = 0 \,, \qquad (14)$$

where

$$\mathbf{U} = \left(\rho, u, p, \rho_1^0, \alpha_1, ..., \rho_{m-1}^0, \alpha_{m-1}, \alpha_{m+1}, ..., \alpha_n \right)^{\mathrm{T}} ;$$

$$\mathbf{B} = \frac{Nu}{r} \left(\rho, 0, \rho c^2, \rho_1^0 G_1, \alpha_1\,(1 - G_1), ..., \rho_{m-1}^0 G_{m-1}, \alpha_{m-1}\,(1 - G_{m-1}), \alpha_{m+1}, ..., \alpha_n \right)^{\mathrm{T}} ;$$

$$\mathrm{A} = \begin{pmatrix}
u & \rho & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\
0 & u & 1/\rho & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\
0 & \rho c^2 & u & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\
0 & \rho_1^0 G_1 & 0 & u & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\
0 & \alpha_1\,(1 - G_1) & 0 & 0 & u & \dots & 0 & 0 & 0 & \dots & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & \rho_{m-1}^0 G_{m-1} & 0 & 0 & 0 & \dots & u & 0 & 0 & \dots & 0 \\
0 & \alpha_{m-1}\,(1 - G_{m-1}) & 0 & 0 & 0 & \dots & 0 & u & 0 & \dots & 0 \\
0 & \alpha_{m+1} & 0 & 0 & 0 & \dots & 0 & 0 & u & \dots & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & \alpha_n & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & u
\end{pmatrix} .$$

Here N is the geometric factor characterizing the symmetry of motion (N = 0, 1, and 2 respectively for planar, cylindrical, and spherical symmetry) and T is the transposition operator. The characteristic equation of system (14) can be obtained from the expression

$$\begin{vmatrix} \xi-u & -\rho & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ 0 & \xi-u & -1/\rho & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ 0 & -\rho c^2 & \xi-u & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ 0 & -\rho_1^0 G_1 & 0 & \xi-u & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ 0 & \alpha_1(G_1-1) & 0 & 0 & \xi-u & \ldots & 0 & 0 & 0 & \ldots & 0 \\ \ldots\ldots & \ldots\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & -\rho_{m-1}^0 G_{m-1} & 0 & 0 & 0 & \ldots & \xi-u & 0 & 0 & \ldots & 0 \\ 0 & \alpha_{m-1}(G_{m-1}-1) & 0 & 0 & 0 & \ldots & 0 & \xi-u & 0 & \ldots & 0 \\ 0 & -\alpha_{m+1} & 0 & 0 & 0 & \ldots & 0 & 0 & \xi-u & \ldots & 0 \\ \ldots\ldots & \ldots\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & -\alpha_n & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & \xi-u \end{vmatrix} = 0\,, \tag{15}$$

where $\xi = dr/dt$. After computing the determinant (15), we have

$$(\xi-u)^{n+m-1}\big(\xi-(u-c)\big)\big(\xi-(u+c)\big)=0\,. \tag{16}$$

From (16), it is seen that the roots of the characteristic equation are real; furthermore, the matrix A can be represented in the form $A = \Omega^{-1}\Lambda\Omega$, where $\Lambda = (\lambda_p\delta_{pk})$ and λ_p are the eigenvalues of the matrix A and δ_{pk} is the Kronecker symbol; consequently, system (14) belongs to a hyperbolic type.

The characteristic relations on the characteristics $\xi = u \pm c$ can be obtained from the determinant (15) in which the last column is replaced by the column vector

$$\begin{pmatrix} -\dfrac{N\rho u\xi_i}{r} - u\dfrac{d\rho}{dt} - \rho\dfrac{du}{dt} \\ -u\dfrac{d\rho}{dt} - \dfrac{1}{\rho}\dfrac{dp}{dt} \\ -u\left(\dfrac{dp}{dt} - c^2\dfrac{d\rho}{dt}\right) \\ -\rho_1^0 G_1\left(\dfrac{Nu\xi_i}{r} + \dfrac{du}{dt}\right) - u\dfrac{du}{dt} \\ -\alpha_1(1+G_1)\left(\dfrac{Nu\xi_i}{r} + \dfrac{du}{dt}\right) - u\dfrac{d\alpha_1}{dt} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ -\rho_{m-1}^0 G_{m-1}\left(\dfrac{Nu\xi_i}{r} + \dfrac{du}{dt}\right) - u\dfrac{du}{dt} \\ -\alpha_{m-1}(1+G_{m-1})\left(\dfrac{Nu\xi_i}{r} + \dfrac{du}{dt}\right) - u\dfrac{d\alpha_{m-1}}{dt} \\ -\alpha_{m+1}\left(\dfrac{Nu\xi_i}{r} + \dfrac{du}{dt}\right) - u\dfrac{d\alpha_{m+1}}{dt} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ -\alpha_n\left(\dfrac{Nu\xi_i}{r} + \dfrac{du}{dt}\right) - u\dfrac{d\alpha_n}{dt} \end{pmatrix}.$$

On the trajectory characteristic $\xi = dr/dt = u$, the relations

$$dp - c^2 d\rho = 0 \,; \quad \frac{d\rho_i}{\rho_i} - \frac{d\rho}{\rho} = 0 \,, \quad \frac{d\alpha_i}{\alpha_i} - (1 - G_i)\frac{d\rho}{\rho} = 0 \,, \quad i = 1, ..., m - 1 \,;$$
$$\frac{d\rho}{\rho} - \frac{d\alpha_j}{\alpha_j} = 0 \,, \quad j = m + 1, ..., n \,, \tag{17}$$

which follow from the differential equations (14) are fulfilled. The characteristics given here and the relations on them can be used for numerical integration of the initial system by the classical method of characteristics.

Numerical Integration of the Model's Equations. We consider planar ($N = 0$) one-dimensional flow of a binary mixture consisting of an ideal gas with an adiabatic exponent γ and the second incompressible fraction. In the case in question, the equation of state of the mixture (8) takes the form

$$\varepsilon = \frac{1}{\rho}\left(\frac{\alpha p}{\gamma - 1} + Z\,(1 - \alpha)\right). \tag{18}$$

Here α is the volume fraction of the gas in the mixture and $Z = \varepsilon_2 \rho_2^0 = \text{const}$. The system of governing equations will be rewritten in vector-matrix form

$$\frac{\partial \mathbf{U}}{\partial t} + \text{A}\,\frac{\partial \mathbf{U}}{\partial r} = 0 \,, \tag{19}$$

where

$$\mathbf{U} = \begin{pmatrix} \rho \\ u \\ p \\ \alpha \end{pmatrix}; \quad \text{A} = \begin{pmatrix} u & \rho & 0 & 0 \\ 0 & u & 1/\rho & 0 \\ 0 & \rho c^2 & u & 0 \\ 0 & \alpha - 1 & 0 & u \end{pmatrix};$$

$c = \sqrt{\dfrac{\gamma p}{\alpha \rho}}$ is the velocity of sound. The expansion $\text{A} = \Omega^{-1}\Lambda\Omega$, where

$$\Omega = \begin{pmatrix} 0 & -\rho c & 1 & 0 \\ -c^2 & 0 & 1 & 0 \\ \frac{1}{\rho} & 0 & 0 & \frac{1}{1-\alpha} \\ 0 & \rho c & 1 & 0 \end{pmatrix}; \quad \Lambda = \begin{pmatrix} u - c & 0 & 0 & 0 \\ 0 & u & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & u + c \end{pmatrix}; \quad \Omega^{-1} = \begin{pmatrix} \frac{1}{2c^2} & -\frac{1}{c^2} & 0 & \frac{1}{2c^2} \\ -\frac{1}{2\rho c} & 0 & 0 & \frac{1}{2\rho c} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{\alpha - 1}{2\rho c^2} & \frac{1-\alpha}{\rho c^2} & 1 - \alpha & \frac{\alpha - 1}{2\rho c^2} \end{pmatrix}$$

holds true for the matrix A.

Figure 1 gives results of calculations of the problem of arbitrary-discontinuity disintegration for the regime of flow with a shock wave moving to the right and a rarefaction wave moving to the left; these results have been obtained by the time $t = 0.4$ sec according to one modified Courant–Isaacson–Rees scheme (see [5]):

$$\frac{\mathbf{U}_i^{k+1} - \mathbf{U}_i^k}{\Delta t} + \left(\Omega^{-1}\Lambda^{-}\Omega\right)_{i+1/2}^{k} \frac{\mathbf{U}_{i+1}^k - \mathbf{U}_i^k}{\Delta r} + \left(\Omega^{-1}\Lambda^{+}\Omega\right)_{i-1/2}^{k} \frac{\mathbf{U}_i^k - \mathbf{U}_{i-1}^k}{\Delta r} = 0 \,, \tag{20}$$

where $\Lambda^{\pm} = \frac{1}{2}\left(\Lambda \pm |\Lambda|\right)$. The parameters of the media before the disintegration are as follows: $p_{0(1)} = 0.5$ MPa, $p_{0(2)} = 0.1$ MPa, $u_{0(1)} = u_{0(2)} = 0$, $\alpha_{10(1)} = \alpha_{10(2)} = 0.9$, $\rho^0_{10(1)} = \rho^0_{10(2)} = 1.19$ kg/m^3, and $\rho^0_{20(1)} = \rho^0_{20(2)} = 1000$

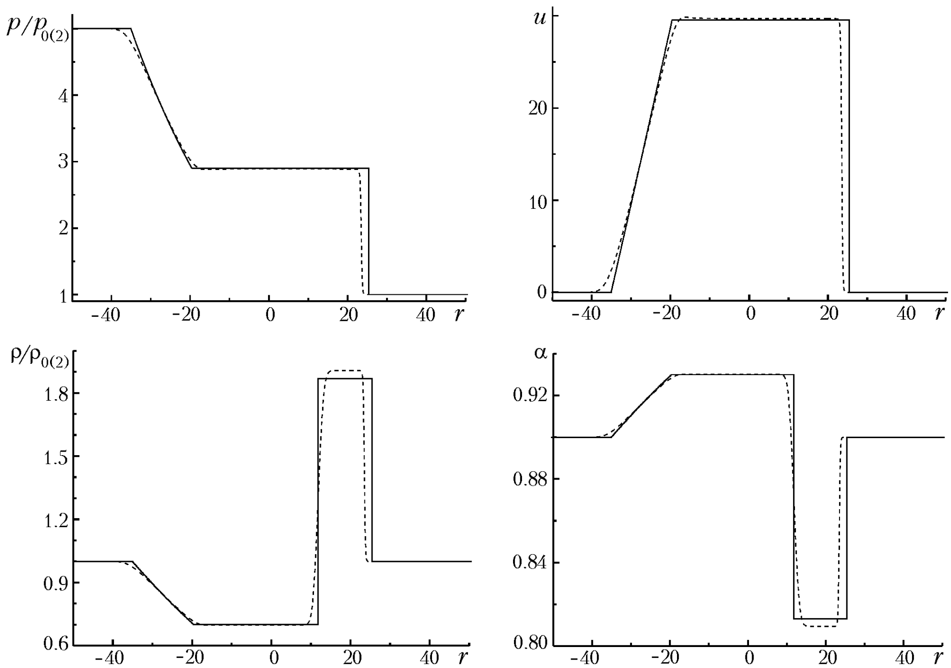

Fig. 1. Dependences of the parameters in discontinuity disintegration on r for t = 0.4 sec: solid curves, exact solution; dashed curves, dependences obtained according to the finite-difference scheme (20).

kg/m^3. The dependences obtained according to the algorithm of exact solution of the Riemann problem are marked by the solid curves in the same figure (see [6]). From Fig. 1, it is clear that the scheme used fairly well calculates rarefaction waves and contact boundaries, not shock waves. When the shock wave is computed, the velocity of its motion and the amplitude differ from the exact values calculated according to the algorithm from [6]. The reason is that the difference scheme approximating the initial system of equations in nondivergent form is nonconservative. We note that not all the model's equations can generally be transformed to a divergent form. This is particularly true of the heat-flux equations. The equations of the model for the binary mixture with one incompressible component can be written in divergent form. Thus, if we introduce the vector $\mathbf{V} = (\rho, \rho u, \rho e, \alpha)^{\mathrm{T}}$, the governing system of equations (19), with account for the equation of state (18), will take the form

$$\frac{\partial \mathbf{V}}{\partial t} + \frac{\partial \Pi(\mathbf{V})}{\partial r} = 0\,, \tag{21}$$

where

$$\Pi(\mathbf{V}) = \begin{pmatrix} \rho u \\ p + \rho u^2 \\ (p + \rho e)\, u \\ -(1-\alpha)\, u \end{pmatrix} = \begin{pmatrix} V_2 \\ \dfrac{\gamma - 1}{V_4}\left[V_3 - (1 - V_4)\, Z - \dfrac{V_2^2}{2V_1}\right] + \dfrac{V_2^2}{V_1} \\ \dfrac{V_2}{V_1}\left\{V_3 + \dfrac{\gamma - 1}{V_4}\left[V_3 - (1 - V_4)\, Z - \dfrac{V_2^2}{2V_1}\right]\right\} \\ -\dfrac{V_2\,(1 - V_4)}{V_1} \end{pmatrix}.$$

Expressions for the pressure and the velocity of sound in new notation will be written as follows:

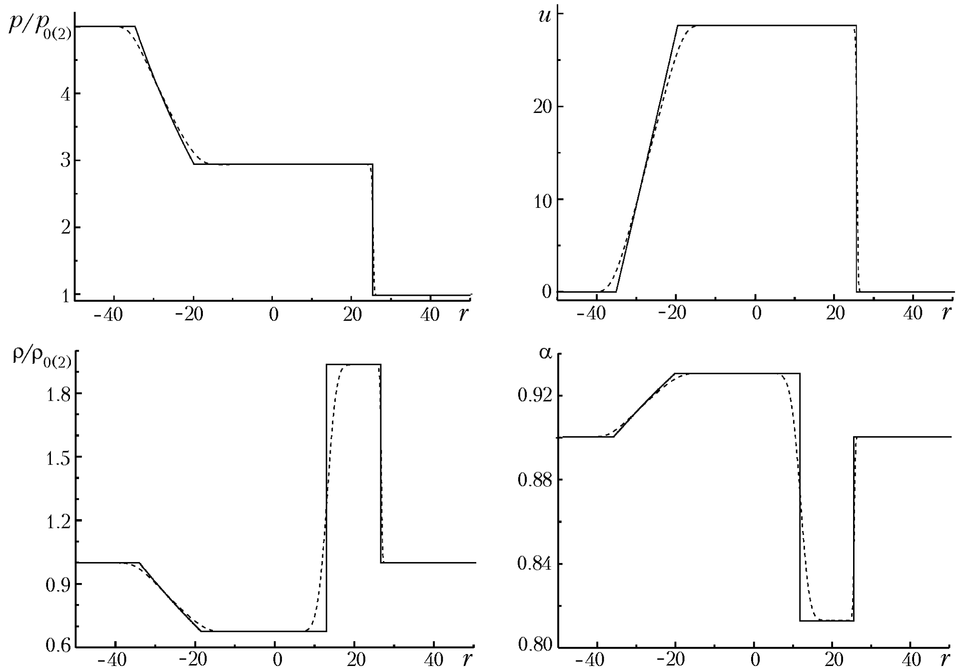

Fig. 2. Dependences of the parameters in discontinuity disintegration on r for t = 0.4 sec: solid curves, exact solution; dashed curves, dependences obtained according to the finite-volume scheme (22)–(23).

$$p = \frac{\gamma - 1}{V_4}\left[V_3 - (1 - V_4)\,Z - \frac{V_2^2}{2V_1}\right], \quad c = \frac{1}{V_1 V_4}\sqrt{\gamma(\gamma - 1)\left[V_1 V_3 - V_1 (1 - V_4)\,Z - \frac{1}{2}\,V_2^2\right]}\,.$$

The finite-volume scheme for system (21) in divergent form has the form

$$\frac{\mathbf{V}_i^{k+1} - \mathbf{V}_i^k}{\Delta t} + \frac{\mathbf{\Pi}_{i+1/2}^k - \mathbf{\Pi}_{i-1/2}^k}{\Delta r} = 0\,. \tag{22}$$

In calculating fluxes through the sides of the cells, we can use the Lax–Friedrichs or Harten–Lax–Van Leer approaches (see [5]). In the first scheme, in expressions (22), we use the relations

$$\mathbf{\Pi}_{l-1/2}^k = \frac{1}{2}\left(\mathbf{\Pi}_{l-1}^k + \mathbf{\Pi}_l^k\right) + \frac{1}{2}\left(\max_p\left(|\lambda_p|\right)\right)_{l-1/2}^k\left(\mathbf{V}_{l-1}^k - \mathbf{V}_l^k\right) = 0 \quad (l = i,\, i+1)\,, \tag{23}$$

in the second scheme, we use the expressions

$$\mathbf{\Pi}_{l-1/2}^k = \begin{cases} \mathbf{\Pi}_{l-1}^k\,, & \text{if } \lambda_1 > 0\,, \\ \dfrac{\lambda_4 \mathbf{\Pi}_{l-1}^k - \lambda_1 \mathbf{\Pi}_l^k + \lambda_1 \lambda_4\,(\mathbf{V}_l^k - \mathbf{V}_{l-1}^k)}{\lambda_4 - \lambda_1}\,, & \text{if } \lambda_1 < 0\,,\ \lambda_4 > 0\,, \\ \mathbf{\Pi}_l^k\,, & \text{if } \lambda_4 < 0\,. \end{cases} \tag{24}$$

Figure 2 gives results of calculations of the problem of arbitrary-discontinuity disintegration, which has been considered earlier; these results have been obtained with the scheme (23) by the time t = 0.4 sec. Analogous data are provided by the method (24). It is seen in the figure that the velocity of movement and the amplitude of the shock wave agree with the exact values. A strong "smearing" of the contact boundary is noteworthy, which is characteristic

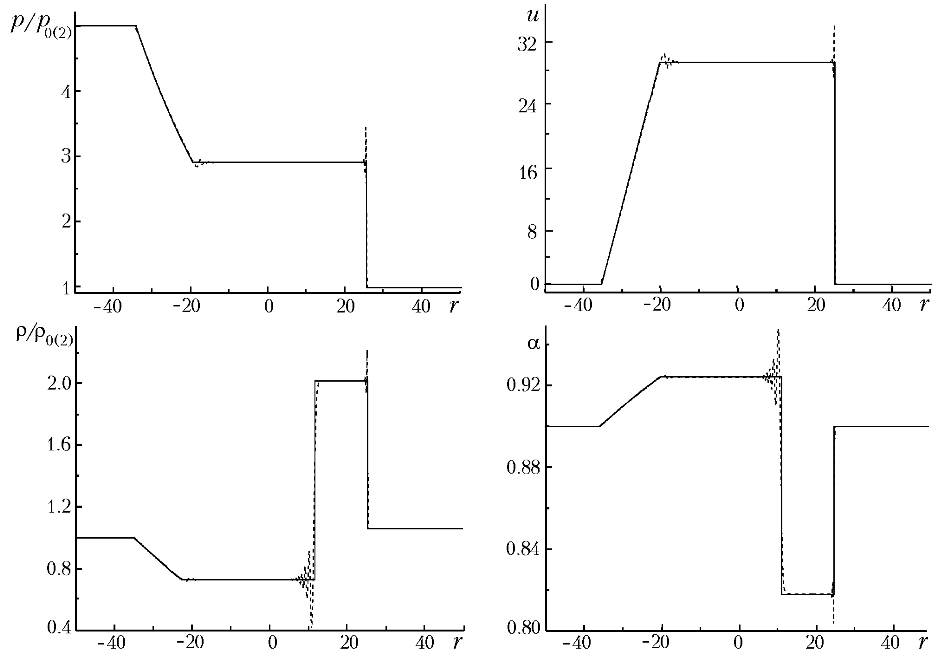

Fig. 3. Dependences of the parameters in discontinuity disintegration on r for t = 0.4 sec: solid curves, exact solution; dashed curves, dependences calculated from the Lax–Wendroff scheme.

of difference schemes of first order of accuracy, among which are the schemes considered above. More accurate results are produced by the methods of higher order of accuracy. For example, Fig. 3 gives results of calculations of the above problem that have been obtained using the Lax–Wendroff scheme of second order of accuracy

$$\mathbf{V}_i^{k+1} = \mathbf{V}_i^k - \frac{\Delta t}{\Delta r}\frac{\Pi_{i+1}^k - \Pi_{i-1}^k}{2} + \frac{1}{2}\left(\frac{\Delta t}{\Delta r}\right)^2 \left[H_{i+1/2}^k\left(\Pi_{i+1}^k - \Pi_i^k\right) - H_{i-1/2}^k\left(\Pi_i^k - \Pi_{i-1}^k\right)\right], \tag{25}$$

where H is the Jacobi matrix ($H_{ij} = \partial\Pi_i/\partial V_j$). We note that the agreement between the numerical and exact solutions is nearly complete, except for the presence of pulsations of parametric values ahead of the contact discontinuity and the shock front in the numerical calculation. We note that limiters of different kinds (see [5]) or certain "smoothing" operators such as those, e.g., in [7], can be introduced into the algorithm of calculation to eliminate "parasitic" oscillations in the solution with the scheme (25). In all the calculations given above, computations have been performed on a uniform grid of 2000 cells with a constant Courant number of 0.7.

Conclusions. The interfractional-interaction forces included in the model of a one-velocity heterogeneous medium ensure the hyperbolic character of the adiabatic approximation and make it possible to obtain a physically consistent flow pattern. Calculations of flows described by the complete system of differential equations (1) which allow for the presence of heat- and mass-exchange processes and physicochemical transformations can be carried out on the basis of the difference schemes used in the work, if the well-known procedure of splitting by physical factors is used.

NOTATION

c, velocity of sound in the mixture, m/sec; c_{*i}, constant of the equation of state, m/sec; $e = \varepsilon + \frac{1}{2}u^2$, specific total energy of the mixture, m^2/sec^2; $\mathbf{f}_{ij}$, density of the force of interfractional interaction between the ith and jth fractions, kg/($m^2\cdot sec^2$); $\mathbf{F}$, density of the mass force, kg/($m^2\cdot sec^2$); J_{ij}, intensity of transformation of the mass from the

ith fraction to the jth fraction in a unit volume of the mixture, kg/(m^3·sec); p, pressure, Pa; Q_{ij}, heat release in a unit time per unit volume of the mixture due to the transformation of the ith fraction to the jth fraction, kg/(m·sec^3); r, space variable, m; R_{ij}, quantity of heat in a unit time per unit volume of the mixture, which goes from the jth fraction to the ith fraction by radiative heat exchange; t, time, sec; T, temperature, K; **u**, velocity vector; $\mathbf{W}_i$, vector of the heat-flux density, kg/sec^3; α_i, volume fraction of the ith component of the mixture; γ_i, constant of the equation of state; ε, specific internal energy of the mixture, m^2/sec^2; ε_i, specific internal energy of the ith fraction, m^2/sec^2; ρ, density of the mixture, kg/m^3; ρ_i^0, true density of the ith fraction, kg/m^3; $\rho_i = \alpha_i\rho_i^0$, reduced density of the ith component, kg/m^3; ρ_{*i}, constant of the equation of state, kg/m^3. Subscripts: 0, in an unperturbed medium; (1) and (2), for the parameters of the mixture to the "left" and to the "right" of the contact discontinuity.

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